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FREQUENCY OF TRANSVERSE VIBRATION OF REGULAR POLYGONAL PLATES WITH A CONCENTRIC, CIRCULAR ORTHOTROPIC PATCH

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1. INTRODUCTION

The present study constitutes a first order approximation treatment of the case where a repair is made of the central circular portion of a vibrating isotropic plate of regular polygonal shape using a material of circularly anisotropic constitutive characteristics. The mathematical model is also valid in the case of a vibrating isotropic plate where the central core has been damaged and it is assumed that this subdomain of the plate acquires polarly orthotropic properties.

The combined physical domain in the z-plane is approximately transformed onto a unit circle with a concentric circle of radius $r_o \ll 1$ in the ζ-plane. Simple co-ordinate functions are used to approximate the fundamental mode shape and the optimized Rayleigh–Ritz method is employed to evaluate the frequency coefficient.

2. APPROXIMATE CONFORMAL MAPPING—VARIATIONAL SOLUTION

The analytic function which maps a regular polygonal shape in the z-plane onto a unit circle in the ζ-plane (Fig. 1) is given by [1]

$$z = f(\zeta) = A_s a_p \int_0^{5} \frac{\mathrm{d}\zeta}{(1+\zeta^s)^{2/s}} = A_s a_p F(\zeta), \quad \zeta = r\mathrm{e}^{\mathrm{i}\alpha}, \tag{1}$$

where A_s is the coefficient which depends on the degree of the polygon [1].

As proposed in previous works the amplitude of the fundamental mode will be approximated by means of co-ordinate functions which do not take into account the azimuthal variation in the ζ-plane. Accordingly the α-dependence will be disregarded. Consequently, the strain energy functional for the transformed isotropic subdomain results [1] in

$$\frac{A_s^2 a_p^2}{D} J_1(W) = \iint_{C_1} \left\{ \frac{1+\nu}{2} \frac{(W'' + W'/r)^2}{|F'(\zeta)|^2} + \frac{1-\nu}{2} \frac{[(W''-W'/r)]G_1 - 2W'H_1]^2 + [(W''-W'/r)G_2 - 2W'H_2]^2}{|F'(\zeta)|^4} \right\} r\,\mathrm{d}r\,\mathrm{d}\alpha, \tag{2}$$

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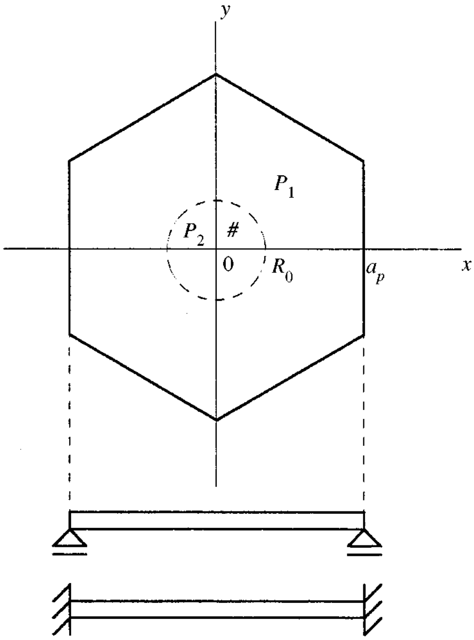

Figure 1. Structural element executing transverse vibrations, considered in the present investigation: $P = P_1 \cup P_2$.

where

$$G_1 + G_2 \mathrm{i} = (\cos 2\alpha - \mathrm{i}\sin 2\alpha)F'(\zeta), \quad H_1 + H_2 \mathrm{i} = (\cos\alpha - \mathrm{i}\sin\alpha)F''(\zeta)$$

On the other hand, the strain energy functional of the orthotropic subdomain in the real plane is given by

$$J_2(W) = \iint_{p_2} \left[D_r W''^2 + D_\theta \left(\frac{W'}{\bar{r}} \right)^2 + 2 D_r \nu_\theta \frac{W' W''}{\bar{r}} \right] \bar{r}\, \mathrm{d}\bar{r}\, \mathrm{d}\theta. \tag{3}$$

Assuming now that $R_o/a_p \ll 1$ it turns out that [1]

$$z_0 \cong A_s a_p \zeta_0 \tag{4}$$

and then

$$r_0 = R_0 / A_s a_p \tag{5}$$

while, for a generic point in P_2, one has

$$\bar{r} = A_s a_p r. \tag{6}$$

Substituting equation (6) into equation (3) one obtains

$$\frac{A_s^2 a_p^2}{D} J_2(W) = \iint_{c_2} \left[\frac{D_r}{D} W''^2 + \frac{D_0}{D} \left(\frac{W'}{r} \right)^2 + 2 \frac{D_r \nu_0}{D} \frac{W' W''}{r} \right] r\, \mathrm{d}r \tag{7}$$

and the complete energy functional results in

$$\frac{A_s^2 a_p^2}{D} J(W) = \frac{A_s^2 a_p^2}{D} [J_1(W) + J_2(W)] - \frac{A_s^4}{16 \tan^4(\pi/s)} \Omega^2 \iint_c W^2 |F'(\zeta)|^2 r\, \mathrm{d}r, \tag{8}$$

where

$$\Omega^2 = \frac{\rho h a^4}{D} \omega^2$$

where a is the side of the polygon. In order to apply the optimized Rayleigh–Ritz method one assumes now

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(r), \tag{9}$$

TABLE 1

Fundamental frequency coefficients of simply supported and clamped isotropic plates of regular ploygonal shape

		Square	Pentagon	Hexagon	Heptagon	Octagon
Simply supported	Present results	20·58	11·21	6·98	5·00	3·71
	Exact	19·74	—	—	—	—
	Reference [3]	19·74	11·01	7·15	5·06	3·79
Clamped	Present results	36·48	19·91	12·83	9·04	6·75
	Reference [3]	35·08	19·71	12·81	9·08	6·78

TABLE 2

Fundamental frequency coefficients of simply supported plates with a concentric circular patch of polar orthotropy

	D_θ/D	$R_0/a_p = 0{\cdot}1$	0·2	0·3	0·4	0·5
Square	0·50	—	—	—	—	—
	0·75	—	—	—	—	—
	1*	20·58	20·58	20·58	20·58	20·58
	1·25	20·61	20·65	20·75	20·84	20·96
	1·50	20·62	20·71	20·89	21·05	21·31
Pentagon	0·50	11·19	11.12	10·98	10·84	10·62
	0·75	11·20	11·17	11·10	11·04	10·94
	1*	11·21	11·21	11·21	11·21	11·21
	1·25	11·23	11·25	11·31	11·36	11·44
	1·50	11·24	11·29	11·39	11·48	11·63
Hexagon	0·50	6·97	6·93	6·83	6·75	6·60
	0·75	6·98	6·96	6·91	6·87	6·81
	1*	6·98	6·98	6·98	6·98	6·98
	1·25	6·99	7·01	7·04	7·08	7·13
	1·50	7·00	7·03	7·10	7·16	7·26
Heptagon	0·50	4·99	4·96	4·89	4·80	4·73
	0·75	5·00	4·98	4·95	4·91	4·88
	1*	5·00	5·00	5·00	5·00	5·00
	1·25	5·01	5·02	5·05	5·08	5·11
	1·50	5·01	5·04	5·08	5·15	5·20
Octagon	0·50	3·70	3·68	3·63	3·56	3·50
	0·75	3·70	3·69	3·67	3·64	3·61
	1*	3·71	3·71	3·71	3·71	3·71
	1·25	3·71	3·72	3·74	3·77	3·79
	1·50	3·72	3·73	3·77	3·82	3·85

*Isotropic plate.

where

$$\varphi_j(r) = 1 - r^{p+j-1} \tag{10}$$

in the case of simply supported plates, and

$$\varphi_j(r) = (1 - r^{p+j-1})^2 \tag{11}$$

when dealing with clamped plates.

The Rayleigh–Ritz method is implemented in a classical, straightforward manner and the lowest eigenvalue, Ω_1, is minimized with respect to p (Rayleigh's optimization parameter).

3. NUMERICAL RESULTS

All calculations were performed for $D_r/D = 1, \nu = \nu_0 = 0{\cdot}30$ and $N = 3$. In the case of an isotropic, simply supported square plate the present calculations yield $\Omega_1 = 20{\cdot}58$ which is 4% higher than the exact eigenvalue ($2\pi^2$). For a clamped isotropic yields $\Omega_1 = 36{\cdot}48$, this eigenvalue being 1% higher than the extremely accurate result available in [2] ($\Omega_1 = 35{\cdot}987$).

TABLE 3

Fundamental frequency coefficients of clamped plates with a concentric circular patch of polar orthotropy

	D_θ/D	$R_o/a_p = 0{\cdot}1$	0.2	0·3	0·4	0·5
Square	0·50	—	—	—	—	—
	0·75	—	—	—	—	—
	1*	36·48	36·48	36·48	36·48	36·48
	1·25	36·53	36·63	36·83	36·99	37·20
	1·50	36·58	36·77	37·12	37·42	37·85
Pentagon	0·50	19·86	19·73	19·46	19·22	18·88
	0·75	19·88	19·82	19·70	19·59	19·43
	1*	19·91	19·91	19·91	19·91	19·91
	1·25	19·93	19·99	20·10	21·19	20·32
	1·50	19·96	20·07	20·26	20·43	20·68
Hexagon	0·50	12·80	12·72	12·54	12·39	12·16
	0·75	12·81	12·78	12·69	12·62	12·52
	1*	12·83	12·83	12·83	12·83	12·83
	1·25	12·85	12·88	12·95	13·01	13·10
	1·50	12·86	12·93	13·06	13·16	13·33
Heptagon	0·50	9·02	8·96	8·84	8·68	8·57
	0·75	9·03	9·01	8·95	8·88	8·83
	1*	9·04	9·04	9·04	9·04	9·04
	1·25	9·06	9·08	9·13	9·19	9·23
	1·50	9·07	9·12	9·20	9·32	9·40
Octagon	0·50	6·74	6·69	6·60	6·48	6·40
	0·75	6·74	6·72	6·68	6·63	6·59
	1*	6·75	6·75	6·75	6·75	6·75
	1·25	6·76	6·78	6·82	6·86	6·90
	1·50	6·77	6·81	6·87	6·96	7·02

*Isotropic plate.

Table 1 depicats a comparison of values of Ω_1 for simply supported and clamped isotropic plates of regular polygonal shape. The agreement is very good from an engineering viewpoint. Table 2 shows values of Ω_1 for simply supported plates of regular polygonal shape with a core of circular anisotropy while Table 3 deals with the clamped situation. The results are obtained as a function of D_θ/D and R_0/a_p. In the case of simply supported and clamped square plates (Tables 2 and 3) the values of Ω_1 determined in the present study turned out to be very high upper bounds for $D_\theta/D < 1$ and they are not included in the table.

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